AD-A147 947 EVOLUTION OF NONLINEAR WAVE GROUPS ON WATER OF SLOWLY-VARYING DEPTH(U) TECHNION - ISRAEL INST OF TECH HAIFA DEPT OF CIVIL ENGINEERING R IUSIM ET AL. OCT 84 1/1
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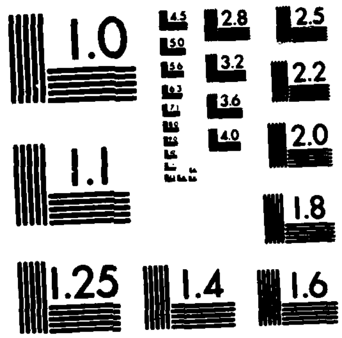

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	
2. GOVT ACCESSION NO.	
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle)	Evolution of nonlinear wave groups on water of slowly varying depth
5. TYPE OF REPORT & PERIOD COVERED	Periodic Report March - Aug 84
6. PERFORMING ORG. REPORT NUMBER	
7. AUTHOR(s)	Ruth Iusim and Michael Stiassnie
8. CONTRACT OR GRANT NUMBER(s)	DAJA37-82-C-0300
9. PERFORMING ORGANIZATION NAME AND ADDRESS	Department of Civil Engineering Technion-Israel Inst. of Sci. & Technology Haifa, Israel
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	61102A-IT161102-BH57-01
11. CONTROLLING OFFICE NAME AND ADDRESS	USARDSG-UK PO Box 65, FPO NY, NY 09510
12. REPORT DATE	October 1984
13. NUMBER OF PAGES	12 (twelve)
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	
15. SECURITY CLASS. (of this report)	Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Water waves; gravity waves, non-linear wave groups; shoaling waves; fourth order equations

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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EVOLUTION OF NONLINEAR WAVE GROUPS ON

WATER OF SLOWLY-VARYING DEPTH

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Availability Codes	
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Contract No. DAJA37-B2-C-0300

Sixth Periodic Report, March-August, 1984

The research reported in this document has been made possible through the support and sponsorship of the U.S. Government through its European Research Office of the U.S. Army

FOURTH ORDER EVOLUTION EQUATIONS FOR SHOALING WAVE GROUPS

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Third order evolution equation for groups of water-waves moving over an uneven bottom were first derived by Djordjevic' and Redekopp (1978). In the present report we carry their derivation one step further, to fourth order, utilizing the REDUCE 2 algebraic manipulator, see Hearn (1973).

We consider the evolution of a uni-directional progressive gravity wave moving along the x axis on the free surface of a homogeneous liquid with depth $h = h(x)$ varying in the direction of the propagation. The effect of surface tension is neglected, so the analysis applies to gravity waves only. The fluid motion is irrotational, thus there exists a velocity potential $\phi(x,z,t)$ satisfying Laplace's equation

$$\phi_{xx} + \phi_{zz} = 0 \qquad (1)$$

where z is the vertical coordinate, and $z = 0$ is the undisturbed free surface.

The boundary condition on the bottom $z = -h(x)$ is

$$\phi_z = -h'(x)\,\phi_x \qquad (2)$$

and the boundary conditions on the free surface $z = \zeta(x,t)$ are the kinematic condition:

$$\phi_z = \zeta_t + \phi_x \zeta_x \tag{3}$$

and the pressure condition:

$$2g\zeta + 2\phi_t + \phi_x^2 + \phi_z^2 = 0 \tag{4}$$

The situation where the depth varies slowly in the direction of propagation is considered, so that properties characterizing the wave will change slowly as well. A small nondimensional parameter ε that measures the slope of the wavy surface is introduced, and we define the new variables:

$$\tau = \varepsilon\left[\int^x \frac{dx}{C_g(\xi)} - t\right] ; \quad \xi = \varepsilon^2 x \tag{5}$$

where C_g is the group velocity.

We suppose that the depth changes on the scale of ε^2 so that $h = h(\xi)$ with the property $h'(\xi) = O(1)$.

The velocity potential and the free surface displacement are expanded as follows:

$$\phi=\phi_o(\xi,\tau,z)+\{\phi_1(\xi,\tau,z)e^{i\theta}+\phi_2(\xi,\tau,z)e^{2i\theta}+\ldots+c.c\} \tag{6}$$

$$\zeta=\zeta_o(\xi,\tau)+\{\zeta_1(\xi,\tau)\ e^{i\theta}+\ \zeta_2(\xi,\tau)e^{2i\theta}+\ldots+c.c\} \tag{7}$$

where $\theta = \int^{x} k(\xi)dx - \omega t$ (8)

and c.c means complex conjugate.

With ε chosen to be small, the functions $\phi_j(\xi,\tau, z)$ and $\zeta_j(\xi,\tau), j > 1$ are expanded formally in powers of ε as follows:

$$\phi_j(\xi,\tau, z) = \sum_{m=j}^{\infty} \varepsilon^m \phi_{mj}(\xi,\tau,z) \quad (9)$$

$$\zeta_j(\xi,\tau) = \sum_{m=j}^{\infty} \varepsilon^m \zeta_{mj} (\xi,\tau) \quad (10)$$

We also assume $\phi_o(\xi,\tau,z) \leqslant 0(\varepsilon)$ (11)

thus only wave induced currents are considered.

Substituting the Fourier series (6) into the Laplace equation (1) we obtain that the zero order potential satisfies:

$$\underline{\underline{\nabla^2 \phi_o(\xi,\tau,z) = 0}} \quad (12)$$

From (11) and (12) it follows that:

$$\phi_{o_{zz}} (\xi,\tau,z) \leqslant 0(\varepsilon^3) \quad (13)$$

Expanding the free surface conditions (3) and (4) around the equilibrium position $z = 0$, substituting eqs. (6) to (10) and looking for the coefficient of e^o, yields respectively:

$$\phi_{o_z} + \varepsilon\zeta_{o_\tau} + 2\varepsilon^3\cdot \mathrm{Re}[\zeta_{11}\bar{\phi}_{21_{zz}} + \zeta_{21}\bar{\phi}_{11_{zz}} + \frac{ki}{C_g}\zeta_{11}\bar{\phi}_{11_\tau} -$$

$$- k^2\phi_{11}\overline{\zeta_{21}} - \frac{ik}{C_g}\zeta_{11}\bar{\phi}_{11_\tau} - k^2\zeta_{11}\bar{\phi}_{21}] + 0(\varepsilon^4) = 0 \tag{14}$$

and

$$\zeta_o = \frac{\varepsilon}{g}\phi_{o_\tau} + 2\mathrm{Re}\,\frac{\varepsilon^2}{g}[i\omega\phi_{11_z}\overline{\zeta_{11}}] - \frac{\varepsilon^2}{g}[k^2\phi_{11}\bar{\phi}_{11} + \phi_{11_z}\bar{\phi}_{11_z}] + 0(\varepsilon^3) \tag{15}$$

where $\phi_o = \phi_o(\xi,\tau,o)$, $\phi_{ij} = \phi_{ij}(\xi,\tau,o)$, and the bar denotes the complex conjugate.

From (14) and (15) it follows that the order of ζ_o is greater or equal to ε^2 and that the order of $\phi_{o_z}(0)$ is greater or equal to ε^3

$$\zeta_o \leqslant 0(\varepsilon^2) \tag{16}$$

and

$$\phi_{o_z}(0) \leqslant 0(\varepsilon^3) \tag{17}$$

The next step of the derivation is similar to that given by Djordjevic' and Redekopp with the difference that we continue the process to fourth order. Substitution of (6) (8) and (9) into Laplace's equation (1) gives:

$\underline{0(\varepsilon)e^{i\theta}:}$
$$\phi_{11} = A(\xi,\tau)\,\frac{\cosh k(z+h)}{\cosh k\,h} \tag{18}$$

$\underline{\underline{0(\varepsilon^2)e^{i\theta}}}$:

$$\phi_{21} = D(\xi,\tau)\ \frac{\cosh k(z+h)}{\cosh k\ h} - \frac{iA_\tau}{C_g}\ \cdot$$

$$\cdot\ \frac{(z+h)\ \sinh k(z+h) - h\sigma\ \cosh k\ (z+h)}{\cosh k\ h} \tag{19}$$

$\underline{\underline{0(\varepsilon^3)e^{i\theta}}}$:

$$\phi_{31} = G(\xi,\tau)\ \frac{\cosh k(z+h)}{\cosh k\ h}\ -$$

$$-\ \frac{i}{2\ \cosh k\ h}\ \{2kh'A + (k'A - \frac{i}{C_g^2}\ A_{\tau\tau})(z+h)\}\cdot$$

$$\cdot\ (z+h)\ \cosh k(z+h) + \frac{i}{\cosh k\ h}\{\sigma(hk)'A\ -$$

$$-\ A_\xi - \frac{ih\sigma}{C_g^2}\ A_{\tau\tau} - \frac{1}{C_g}\}\ D_\tau\cdot\ (z+h)\ \sinh k(z+h) \tag{20}$$

$\underline{\underline{0(\varepsilon^4)e^{i\theta}}}$:

$$\phi_{41} = M(\xi,\tau)\ \frac{\cosh k(z+h)}{\cosh k\ h}\ +$$

$$+[\ \frac{i}{6C_g^3}\ A_{\tau\tau\tau} - \frac{k'}{2C_g}\ A_\tau]\ [(z+h)^3]\ \frac{\sinh k(z+h)}{\cosh k\ h}\ -$$

$$-\ \frac{kh'}{C_g}\ A_\tau(z+h)^2\ \frac{\sinh k(z+h)}{\cosh k\ h}\ +$$

$$+\{\frac{1}{2kC_g}\{(kh')\,[3\sigma-2kh(2\sigma^2-1)] + kh'\sigma - hk'\sigma - 2kh\sigma\cdot\frac{C_g'}{C_g}$$

$$+\frac{2\sigma^2}{1-\sigma^2}\,\frac{k'}{k}\}\,A_\tau + \frac{h\sigma}{C_g}A_{\tau\xi} - \frac{i\sigma^2}{1-\sigma^2}\frac{k'}{k}D - iD_\xi - \frac{iG_\tau}{C_g}\}\cdot$$

$$\cdot(z+h)\,\frac{\sinh k(z+h)}{\cosh k\,h} + \{\frac{1}{4kC_g}\,[\frac{-4\sigma^2}{1-\sigma^2}k' + 2kh\sigma k' + 2k\frac{C_g'}{C_g}]\,A_\tau$$

$$-\frac{ih\sigma}{2C_g^3}A_{\tau\tau\tau} - \frac{A_{\xi\tau}}{C_g} - \frac{D_{\tau\tau}}{2C_g^2} - \frac{ik'}{2}D\}\quad(z+h)^2\,\frac{\cosh k(z+h)}{\cosh k\,h} +$$

$$+\{-\frac{h'}{C_g}(1-kh\sigma)A_\tau - ikh'D\}\;(z+h)\,\frac{\cosh k(z+h)}{\cosh k\,h} \tag{21}$$

$\underline{0(\varepsilon^2)e^{2i\theta}}$:

$$\phi_{22} = F(\xi,\tau)\,\frac{\cosh 2k(z+h)}{\cosh 2\,k\,h} \tag{22}$$

$\underline{0(\varepsilon^3)e^{2i\theta}}$:

$$\phi_{32} = E(\xi,\tau)\,\frac{\cosh 2k(z+h)}{\cosh 2\,kh} - \frac{iF_\tau}{C_g}\cdot$$

$$\cdot(z+h)\,\sinh\frac{2k(z+h)}{\cosh 2kh} \tag{23}$$

Substitution of eq (6) to (10) and eqs. (18) to (23) into the expansion around z = 0 of the free surface conditions (3) and (4) gives:

$0(\varepsilon)e^{i\theta}$:

$$\zeta_{11} = \frac{i\omega}{g} A \tag{24}$$

and

$$\omega^2 = g\ k\ \sigma \tag{25}$$

where $$\sigma = tgh(kh) \tag{26}$$

$0(\varepsilon^2)e^{i\theta}$:

$$C_g = \frac{g}{2\omega} [\sigma + kh(1-\sigma^2)] \tag{27}$$

and

$$g\zeta_{21} = i\omega D + A_\tau \tag{28}$$

$0(\varepsilon^2)e^{2i\theta}$:

$$g\zeta_{22} = -\frac{k^2}{2} \frac{(3-\sigma^2)}{\sigma^2} A^2 \tag{29}$$

and

$$\omega F = \frac{3}{4} ik^2 \frac{(1-\sigma^4)}{\sigma^2} A^2 \tag{30}$$

$0(\varepsilon^3)e^{i\theta}$:

$$\varepsilon^2\mu_1 A + \varepsilon^3\mu_2 A_\xi + \varepsilon^3\mu_3 A_{\tau\tau} + \varepsilon^2\mu_4 A\phi_{o_\tau} +$$

$$+ \varepsilon^3\mu_5|A|^2 A = 0 \tag{31}$$

where

$$\mu_1 = -i\{ \frac{\sigma^2}{1-\sigma^2} \frac{k'}{k} [\sigma + hk(1-\sigma^2)] + (hk)'\} \tag{32}$$

$$\mu_2 = -\frac{2\omega i C_g}{g} \quad (33)$$

$$\mu_3 = \frac{1}{g}\left[1-\frac{gh}{C_g^2}(1-kh\sigma)(1-\sigma^2)\right] \quad (34)$$

$$\mu_4 = \frac{k^2}{g}\left[2\frac{C_p}{C_g}+(1-\sigma^2)\right]; \quad C_p = \omega/k \quad (35)$$

$$\mu_5 = \frac{k^4}{2g}\left[\frac{9}{\sigma^2} - 12 + 13\sigma^2 - 2\sigma^4\right] \quad (36)$$

and

$$g\zeta_{31} = i\omega G + \left(1+\frac{\omega h\sigma}{C_g}\right)D_\tau + \left(\frac{i\omega k\sigma}{g} - \frac{ik}{C_g}\right)\phi_{o_\tau} A +$$

$$+ \frac{i\omega h^2}{2C_g^2}(2\sigma^2-1)A_{\tau\tau} + h\omega\sigma A_\xi -$$

$$- \frac{ik^4}{2\omega\sigma^2}\{3-9\sigma^2+11\sigma^4 - 2\sigma^6\}A^2\bar{A} +$$

$$+ \frac{h\omega}{2}[2kh' + k'h - 2\sigma^2(kh)']A \quad (37)$$

$\underline{0(\varepsilon^3)e^{2i\theta}:}$

$$\frac{4\sigma^3}{1+\sigma^2}E = \frac{k}{2\omega C_g}[3kh(3\sigma^4+2\sigma^2-5)+9\sigma(1-\sigma^2)]AA_\tau$$

$$+ \frac{6ik\omega}{g}(1-\sigma^2)AD \quad (38)$$

and

$$g\zeta_{32} = \frac{ik}{4C_g\sigma^3}[-12kh(1-\sigma^2)-4\sigma^3+12\sigma]AA_\tau$$

$$- \frac{k^2}{\sigma^2}(3-\sigma^2)AD \quad (39)$$

$\underline{\underline{O(\varepsilon^4)e^{i\theta}}}$:

$$\varepsilon^4\mu_1 D+\varepsilon^4\mu_2 D_\xi +\varepsilon^4\mu_3 D_{\tau\tau}+\varepsilon^3\mu_4 D\phi_{o_\tau} +\varepsilon^4\cdot 2\mu_5|A|^2 D +$$

$$+\varepsilon^4\mu_5 A^2\bar{D} +\varepsilon^4\lambda_1 A_\tau +\varepsilon^4\lambda_2 A_{\tau\tau\tau} +\varepsilon^4\lambda_3|A|^2 A_\tau+\varepsilon^4\lambda_4 A^2\bar{A}_\tau+$$

$$+\varepsilon^4\lambda_5 A_{\xi\tau}+\varepsilon^3\lambda_6 A\phi_{o_{\tau\tau}} +\varepsilon^3\lambda_7 A_\tau\phi_{o_\tau}+\varepsilon^3\lambda_8 A\phi_{o_\xi} +$$

$$+\varepsilon\lambda_9 A\phi_{o_z} = 0 \tag{40}$$

where $\mu_1,\ldots,\mu_5$ are given by (32) to (36) and

$$\lambda_1 = -\frac{h'g^2}{4\omega^2 C_g^3}\,(2h^3k^3\sigma^7-6h^3k^3\sigma^5+6h^3k^3\sigma^3- 2h^3k^3\sigma -$$

$$5h^2k^2\sigma^6+9h^2k^2\sigma^4-5h^2k^2\sigma^2+ h^2k^2+ 4hk\sigma^5 -$$

$$-4hk\sigma^3 - \sigma^4 + \sigma^2) \tag{41}$$

$$\lambda_2 = \frac{ih^2(1-\sigma^2)}{3C_g^3}\,[-3\sigma +kh(3\sigma^2-1)] \tag{42}$$

$$\lambda_3 = \frac{ik^3}{2gC_g\sigma^3}\{7\sigma^7-47\sigma^5+48\sigma^3-36\sigma+kh[-7\sigma^8+32\sigma^6-25\sigma^4-18\sigma^2+18]\} \tag{43}$$

$$\lambda_4 = \frac{ik^3}{2gC_g}\{-\sigma^4+5\sigma^2+kh\sigma(1-\sigma^2)^2\} \tag{44}$$

$$\lambda_5 = \frac{-2h(1-\sigma^2)}{C_g}(1-kh\sigma) \tag{45}$$

$$\lambda_6 = \frac{i\omega k\sigma}{g^2} - \frac{2ik}{gC_g} - \frac{i\omega}{gC_g^2} \tag{46}$$

$$\lambda_7 = -\frac{ik}{gC_g}\{\frac{2C_p}{C_g} + 2[1-kh\sigma][1-\sigma^2] + 2\} \tag{47}$$

$$\lambda_8 = \frac{2\omega}{g}k \tag{48}$$

$$\lambda_9 = -\frac{i\omega k\sigma}{g} \tag{49}$$

Combining eqs. (31) and (40) and introducing the new variable

$$A_1 = \varepsilon A + \varepsilon^2 D \tag{50}$$

one finally obtains the modification to fourth order of the cubic Schrödinger equation

$$\begin{aligned}&\varepsilon^2\mu_1 A_1 + \varepsilon^2\mu_2 A_{1\xi} + \varepsilon^2\mu_3 A_{1_{\tau\tau}} + \varepsilon\mu_4 A_1\phi_{o_\tau} + \mu_5|A_1|^2A_1 + \\ &+ \varepsilon^3\lambda_1 A_{1_\tau} + \varepsilon^3\lambda_2 A_{1_{\tau\tau\tau}} + \varepsilon\lambda_3|A_1|^2A_{1_\tau} + \varepsilon\,\lambda_4 A_1^2\bar{A}_{1_\tau} + \\ &+ \varepsilon^3\lambda_5 A_{1_{\xi\tau}} + \lambda_6\varepsilon^2 A_1\phi_{o_{\tau\tau}} + \varepsilon^2\lambda_7 A_{1_\tau}\phi_{o_\tau} + \lambda_8\varepsilon^2 A_1\phi_{o_\xi} + \lambda_9 A_1\phi_{o_z} = 0\end{aligned} \tag{51}$$

On the other hand substitution of (18) and (24) into (14) and (15) and elimination of ζ_o, yields the following equation for ϕ_o:

$$\phi_{o_z} + \frac{\varepsilon^2 \phi_{o_{\tau\tau}}}{g} = \varepsilon^3 \frac{k^2}{g} \left[\frac{2C_p}{C_g} + (1-\sigma^2)\right] \left(|A|^2\right)_\tau + O(\varepsilon^4) \tag{52}$$

which can be written to the same order of accuracy as

$$\phi_{o_z} + \frac{\varepsilon^2 \phi_{o_{\tau\tau}}}{g} = \varepsilon \frac{k^2}{g} \left[\frac{2C_p}{C_g} + (1-\sigma^2)\right] \left(|A_1|^2\right)_\tau \tag{53}$$

Laplace's equation (12) and the bounadry conditions (2) at z= -h and eqs. (51), (53) at z = 0 form the system of equations from which A_1 and ϕ_o can be determined.

For the case of infinitely deep water, the terms $\lambda_6 \cdot \varepsilon^2 A_1 \phi_{o_{\tau\tau}}$, $\lambda_7 \varepsilon^2 A_{1_\tau} \phi_{o_\tau}$, and $\lambda_8 \varepsilon^2 A_1 \phi_{o_\xi}$ in (53) are $O(\varepsilon^5)$, and equations (53) and (51) become respectively:

$$\phi_{o_z} = \varepsilon \frac{4k^2}{g} \left(|A_1|^2\right)_\tau \tag{54}$$

and

$$-\varepsilon^2 \cdot \frac{2\omega i C_g}{g} A_{1_\xi} + \varepsilon^2 \frac{A_{1_{\tau\tau}}}{g} + \frac{2\omega k}{g C_g} A_1 \phi_{o_\tau} +$$

$$+ \frac{4k^4}{g} |A_1|^2 A_1 + \varepsilon\left[- \frac{14ik^3}{gC_g} |A_1|^2 A_{1_\tau} + \frac{2ik^3}{gC_g} A_1^2 \bar{A}_{1_\tau}\right]$$

$$- \frac{i\omega k}{g} A_1 \phi_{o_z} = 0 \tag{55}$$

The last two equations are identical to those given in K.B. Dysthe (1980).

Note that the following typographical errors were found in Dysthe's paper:

His equations (2.17) and (2.19) should be written as follows:

$$\Gamma = 4k^4|A|^2 + 8ik^3(AA_x^* - A^*A_x) - 4ik^3|A|_x^2 + 2\omega k(\bar{\phi}_x - i\bar{\phi}_z)$$

(Dysthe 2.17)

$$2i(A_t + \tfrac{1}{2}A_x) + \tfrac{1}{2}A_{yy} - \tfrac{1}{4}A_{xx} - A|A|^2 = -\frac{1}{8}i(6A_{xyy} - A_{xxx}) +$$

$$+ \frac{3}{2}iA(AA_x^* - A^*A_x) - \tfrac{1}{2}|A|^2A_x + A(\bar{\phi}_x - i\bar{\phi}_z) \qquad \text{(Dysthe 2.19)}$$

REFERENCES

1. Djordevic', V.D., and Redekopp, L.G. 1978, On the development of packets of surface gravity waves moving over an uneven bottom, J. of Applied Mathematics and Physics, 29, 950-62.
2. Dysthe, K.B., 1980, Note on a modification of the nonlinear Schrödinger equation for application to deep water waves, Proc. R. Soc. A369, 105-14.
3. Hearn, A.C. 1973, REDUCE 2 User's Manual, University of Utah.

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